

ON NEARLY PLANAR ORBITS IN STELLAR SYSTEMS

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Abstract. The purpose of the contribution is a discussion of nearly planar orbits characteristic for stellar discs since then a star remains always in the vicinity of the plane of symmetry.

INTRODUCTION

Though orbits of stars belonging to stellar discs are usually thought to be nearly circular in the plane of symmetry, the star motions perpendicular to that plane are also present and, besides, projected onto the plane the orbits are not always of very low eccentricity. It has been noticed that the highest distance to the plane of symmetry is an increasing function of that to the symmetry axis. As this property follows the law of specific (per unit mass) action conservation, it has been usually attributed to a small ratio of periods – the one of perpendicular motions to the one of motions in the plane. However, it is rather well known that at least at the Sun these two periods, though not equal, have the same order of magnitude. The cause should be sought in another property, which is followed by the question of orbit stability.

MATHEMATICAL BACKGROUND

The Lagrange equations are (near $z = 0$)

$$\ddot{R} - \frac{J_z^2}{R^3} = \frac{\partial \Pi(0)}{\partial R} \quad (1a),$$
$$\ddot{z} = -\kappa_z^2(R)z \quad (1b);$$

Π – potential, J_z - component of specific angular momentum which is conserved,

$$\kappa_z^2(R) = -\frac{\partial^2 \Pi}{\partial z^2}(R, 0) \quad (2).$$

Equation (1a) yields $R_p \leq R \leq R_a$, also

$$R_m = \frac{R_p + R_a}{2}, e = \frac{R_a - R_p}{R_p + R_a}.$$

The eccentricity is not too low, therefore

$$\kappa_z^2(R) = \kappa_z^2(R_m) + \frac{d\kappa_z^2}{dR}(R_m)(R - R_m) \quad (3).$$

In $z = 0$ it is assumed $\kappa_z^2(R) \propto \rho(R, 0)$ (Poisson equation), $\rho(R, 0) \propto \exp(-R/R_d)$, it follows

$$\frac{d\kappa_z^2}{dR}(R_m) = -\frac{\kappa_z^2(R_m)}{R_d}.$$

On the basis of the two last expressions the extremal values of κ_z^2 are

$$\kappa_z^2(R_a) = \kappa_z^2(R_m) \left(1 - \frac{eR_m}{R_d}\right), \kappa_z^2(R_p) = \kappa_z^2(R_m) \left(1 + \frac{eR_m}{R_d}\right) \quad (4).$$

RESULTS

The period P_z corresponding to κ_z is comparable to P_R (between R_a and R_p and backwards) at any R . For this reason during the motion from $-|z|_{max}$ to $|z|_{max}$ and backwards the variations in R are not negligible. Nevertheless, the specific action is conserved. As seen from equations (1b) and (3) the acceleration at a given z is always the same at the same R . As a consequence $|z|_{max}(R_a)$ will be the largest, that for R_p the smallest (equation (4)) and vice versa for the speeds $|\dot{z}|(R_a, 0)$ and $|\dot{z}|(R_p, 0)$. Since at any R it is $\dot{z}^2(R, 0) = \kappa_z^2(R)|z|_{max}^2$, it will be $\dot{z}^2(R, 0) \propto \kappa_z(R)$, $|z|_{max}^2(R) \propto \kappa_z(R)^{-1}$, i. e. specific action conservation.

The question of stability is rather complicated, but at the first glance it is seen that, at least for moderate R_m , stability requires lower eccentricities. Then, $|z|_{max}(R)$ and $|\dot{z}|(R, 0)$ remain within sufficiently low upper limits, where they obey the law of specific action conservation.